

Date:

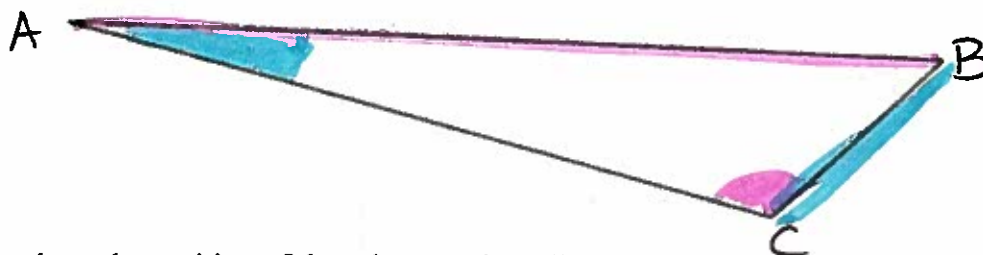
Period: 5

Relating Sides and Angles of a Triangle // Triangle Inequality Theorem

Lesson Objective

INBAT use sides and angles of Δ s to determine relative size.

Use the obtuse triangle below. Color the largest angle and the largest side of the triangle with the SAME color. Then, color the smallest angle and the smallest side of the triangle with a DIFFERENT color



What do you notice about the position of these large and small angles/sides?

larger angle / side opposite / across from each other

These relationships are true for ALL triangles.

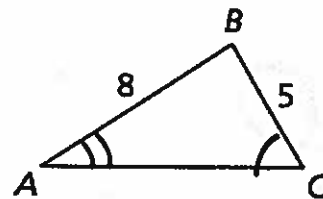
same w/ small
LS and sides

These relationships can help you decide whether a particular arrangement of side lengths and angle measures in a triangle may be possible.

Triangle Longer Side Theorem

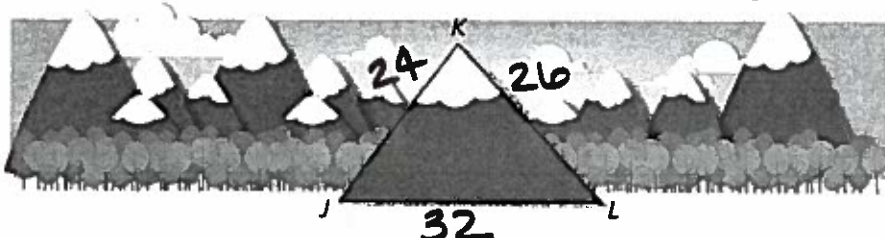
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

$$AB > BC, \text{ then } \dots m\angle C > m\angle A$$



Example:

3. You are constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is about 32 feet long, the left slope is about 24 feet long, and the right slope is about 26 feet long. List the angles of ΔJKL in order from smallest to largest.

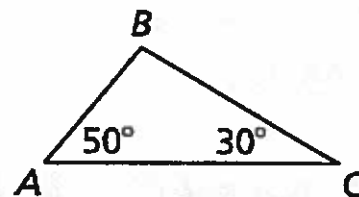


$$m\angle L < m\angle J < m\angle K$$

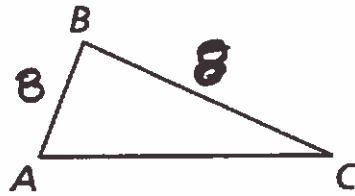
Triangle Larger Angle Theorem

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

$$m\angle A > m\angle C, \text{ then } \dots BC > AB$$



PROOF: Given: $m\angle A > m\angle C$
 Prove: $BC > AB$



Temp. assume

① $BC < AB$ and

② $BC = AB$

① If $BC < AB$, then

$m\angle A < m\angle C$ by Δ longer side thm. But we were given $m\angle A > m\angle C$.

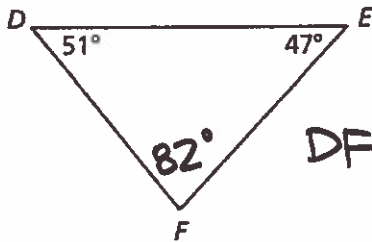
② If $BC = AB$, then

$m\angle A = m\angle C$ by if $\Delta \rightarrow \Delta$

But we were given $m\angle A > m\angle C$.

Example

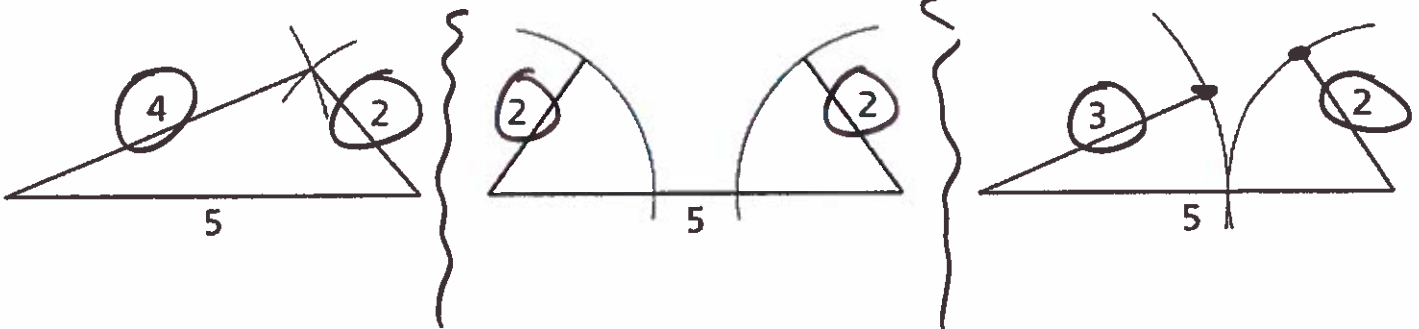
4. List the sides of $\triangle DEF$ in order from shortest to longest.



$DF < EF < DE$

Both assumptions led to contradicting statements, so $BC < AB$ and $BC = AB$ cannot both be true. So, $BC > AB$.

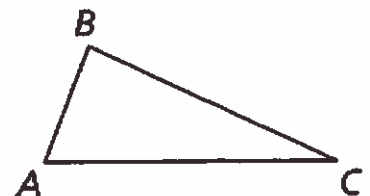
Not every group of three segments can be used to form a triangle the lengths of the segments must fit a certain criteria. For example, three attempted triangle constructions using segments with given lengths are shown below. Only the first group of segments forms a triangle...why?



Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle

is greater than the length of the third side.



$$AB + BC > AC \quad \text{AND} \quad AB + AC > BC \quad \text{AND} \quad BC + AC > AB$$

Example:

5. Decide whether it is possible to construct a triangle with the given side lengths...

a. 4 ft, 9 ft, 10 ft

b. 8 m, 8 m, 18 m

c. 5 cm, 7 cm, 12 cm

$$4 + 9 > 10 \checkmark$$

$$9 + 10 > 4 \checkmark$$

$$10 + 4 > 9 \checkmark$$

YES Δ

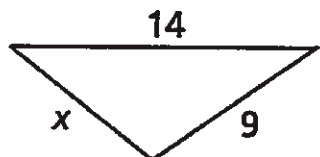
$$8 + 8 > 18 \checkmark$$

NOT Δ

$$5 + 7 > 12 \checkmark$$

NO Δ

6. A triangle has one side length of 14 and another side length of 9. Describe the possible lengths of the third side.



If x is biggest

$$14 + 9 > x$$

$$23 > x$$

If x is smallest...

$$x + 9 > 14$$

$$x > 5$$

$$5 < x < 23$$

Think About It... Is it possible to decide if three side lengths form a triangle without checking all three inequalities shown in the Triangle Inequality Theorem? Explain.

yes! since the largest side is already longer than either of the 2 smaller sides, the sum of the longer and one of the smaller will already be greater than the second smaller side. So, you only need to check if the sum of the 2 smaller sides is greater than the longer side.

